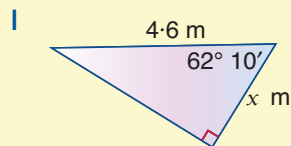
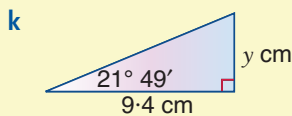
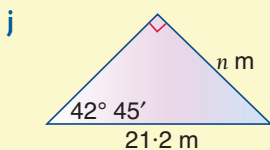
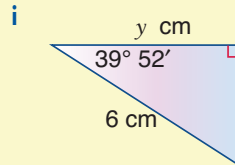
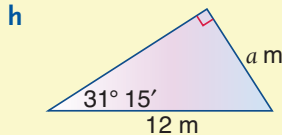
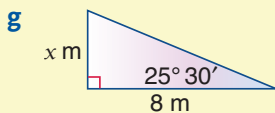
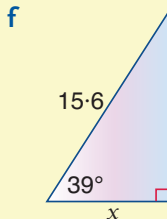
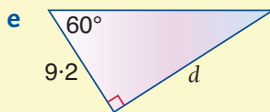
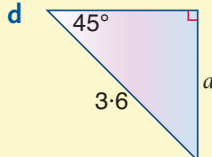
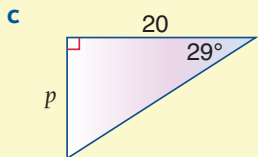
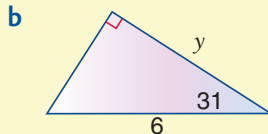
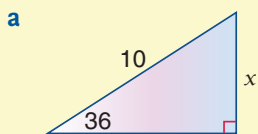
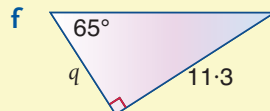
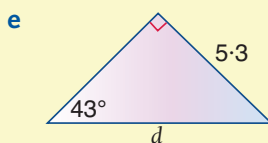
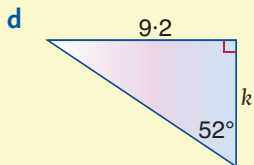
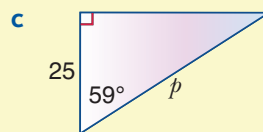
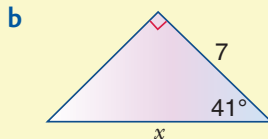
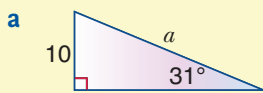


# Exercise 13:05

**1** Find the value of the pronumeral in each triangle, correct to 1 decimal place.



**2** Determine the value of each pronumeral, correct to 1 decimal place.



### Using trigonometry to find side lengths

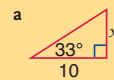
**1** Find  $x$  correct to 1 decimal place:

**a**  $\frac{x}{8} = \sin 15^\circ$     **b**  $\frac{y}{6.8} = \tan 38^\circ \dots$

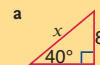
**2** Find  $x$  correct to 1 decimal place:

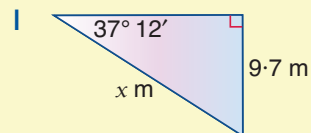
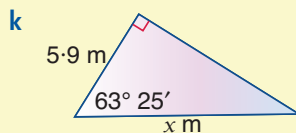
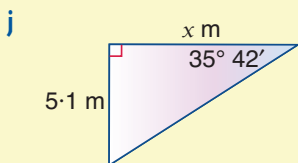
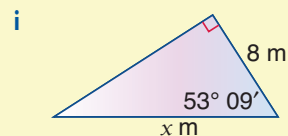
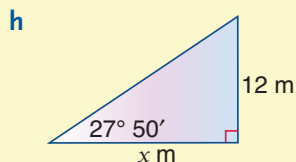
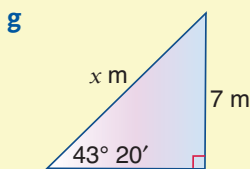
**a**  $\frac{15}{x} = \cos 40^\circ \dots$

**3** In each of the following state which trig. ratio needs to be used to find  $x$  and then find it correct to 1 decimal place.

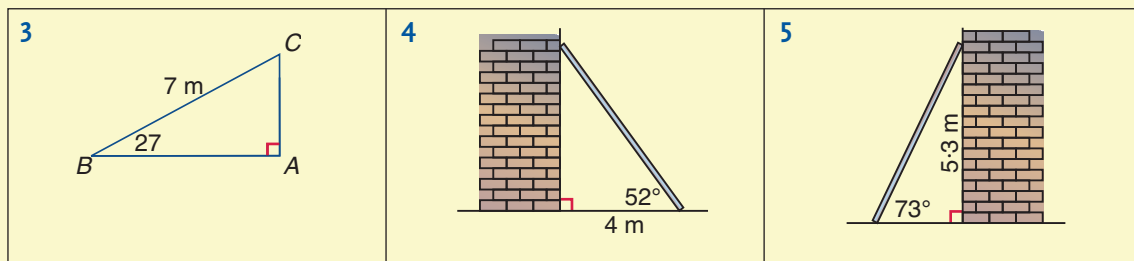


**4** In each of the following state which trig. ratio needs to be used to find the length of the hypotenuse and then find it correct to 1 decimal place.

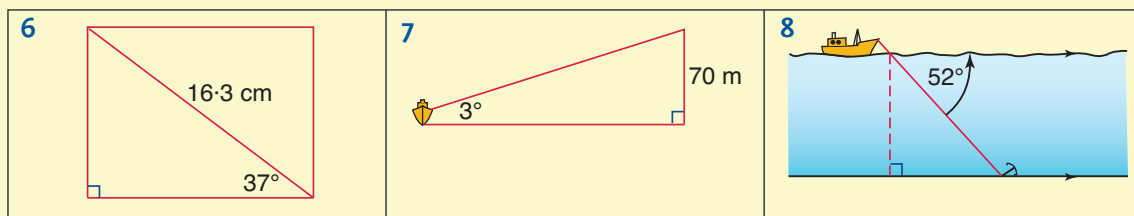




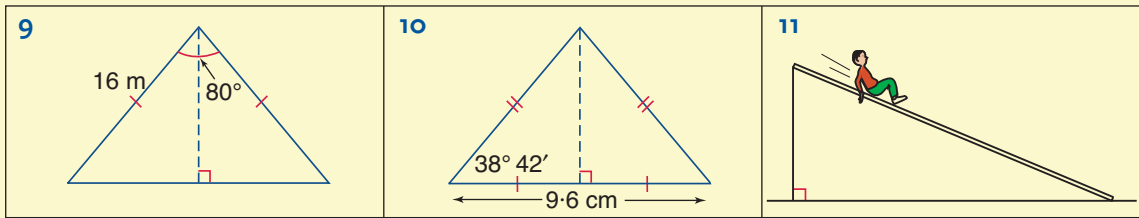
For questions **3** to **11** the diagrams relate to the questions below them.



- 3** Find out everything you can about the triangle.
- 4** A ladder leans against a wall so that the angle it makes with the ground is  $52^\circ$  and its base is  $4$  m from the wall. How far does the ladder reach up the wall (to the nearest centimetre)?
- 5** A ladder leaning against a wall reaches  $5.3$  m up the wall when the angle between the ground and the ladder is  $73^\circ$ . How long, to the nearest centimetre, is the ladder?



- 6** The diagonal of a rectangle is  $16.3$  cm long and makes an angle with the longest side of  $37^\circ$ . Find the length of the rectangle, to the nearest centimetre.
- 7** A ship out at sea observes a lighthouse on the top of a  $70$  m cliff at an angle of  $3^\circ$ . How far out to sea is the ship (to the nearest metre)?
- 8** A boat is anchored in a river that is  $3.2$  m deep. If the anchor rope makes an angle of  $52^\circ$  with the surface of the water, how long is the rope from the surface of the water? (Answer to the nearest centimetre.)



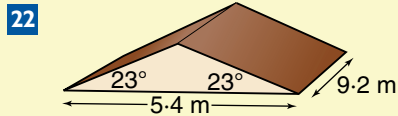
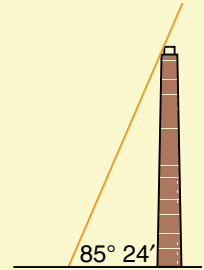
- 9** The equal sides of an isosceles triangle are 16 metres long and the apex angle is  $80^\circ$ . Find, to the nearest centimetre, the length of the base.
- 10** The base of an isosceles triangle is 9.6 cm long and each of the base angles is  $38^\circ 42'$ . Find the length of each of the equal sides. (Answer correct to 3 significant figures.)
- 11** If the length of a child's slippery-dip is 3.4 m and one end makes an angle of  $38^\circ 42'$  with the ground, how high above the ground is the other end? (Answer to the nearest centimetre.)

For questions **12** to **20**, draw a diagram first!

- 12** a In  $\triangle ABC$ ,  $\angle A = 90^\circ$ ,  $\angle B = 63^\circ 25'$  and  $BC = 6$  m. Find  $AC$ , correct to the nearest centimetre.  
 b In  $\triangle XYZ$ ,  $\angle Z = 90^\circ$ ,  $\angle X = 42^\circ 34'$  and  $XZ = 9.2$  m. Find  $YZ$ , correct to the nearest centimetre.  
 c In  $\triangle ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = 52^\circ$  and  $AB = 2.7$  cm. Find  $AC$ , to 1 decimal place.  
 d In  $\triangle XYZ$ ,  $\angle X = 90^\circ$ ,  $\angle Y = 31^\circ 20'$  and  $XZ = 10.3$  cm. Find  $XY$ , to 1 decimal place.
- 13** The diagonal of a square is 21.2 cm. Find the length of each side (to the nearest millimetre).
- 14** Find the length of the diagonal of a rectangle if the length of the rectangle is 7.5 cm and the diagonal makes an angle of  $25^\circ$  with each of the longer sides. (Answer correct to the nearest millimetre.)
- 15** Find the length of a rectangle if its diagonal is 34 cm long and the angle the diagonal makes with the length is  $27^\circ 50'$ . (Answer correct to the nearest centimetre.)
- 16** Find the base of an isosceles triangle if the height is 8.2 cm and the base angles are each  $39^\circ$ . (Answer correct to the nearest millimetre.)
- 17** When the altitude of the sun is  $51^\circ 47'$ , a vertical stick casts a shadow 45 cm long. How high, to the nearest millimetre, is the stick?
- 18** A painting is hung symmetrically by means of a string passing over a nail with its ends attached to the upper corners of the painting. If the distance between the corners is 55 cm and the angle between the two halves of the string is  $105^\circ$ , find the length of the string, correct to the nearest millimetre.
- 19** The vertical rise from the bottom to the top of a track that slopes uniformly at  $6^\circ 54'$  with the horizontal is 36 m. Find, to 1 decimal place, the length of the track.

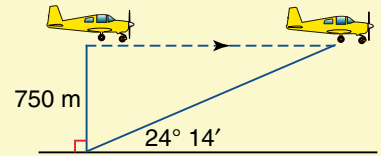
**20** A road rises steadily at an angle of  $6^\circ 45'$ . What will be the vertical rise of the road for a horizontal distance of 300 m? (Answer correct to the nearest metre.)

**21** At noon a factory chimney casts a shadow when the sun's altitude is  $85^\circ 24'$ . If the chimney is 65 m high, what is the length of the shadow, to the nearest centimetre?



Calculate the sloping area of this roof that needs to be tiled, given that the width of the roof is 5.4 m and its length is 9.2 m. Each roof section is pitched at an angle of  $23^\circ$ . (Answer correct to the nearest square metre.)

**23** A plane is flying at an altitude (height) of 750 metres. A boy on the ground first observes the plane when it is directly overhead. Thirty seconds later, the angle of elevation of the plane from the boy is  $24^\circ 14'$ .



**a** Through what distance did the plane fly in 30 seconds, to the nearest metre?

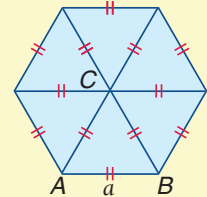
**b** Calculate the speed of the plane in km/h, correct to 3 significant figures.

**24** Calculate the area of a right-angled triangle that has a hypotenuse 8 cm long and an angle of  $50^\circ$ .

**25** A regular hexagon of side  $a$  units is made by joining six equilateral triangles together, as shown in the diagram.

We want to find a formula for the area of the hexagon in terms of its side length,  $a$ .

Consider the area of *one* of the equilateral triangles.



**a** Using the exact trig. ratios on page 469, find the exact length of  $DC$ .

**b** What is the area of  $\triangle ABC$ ?

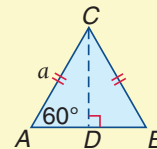
**c** What is the area of a hexagon of side  $a$  units?

**d** Find the area of a hexagon with a side length of:

**i** 2 cm

**ii** 5 cm

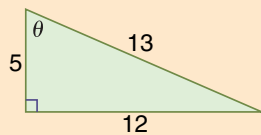
**iii** 10 cm



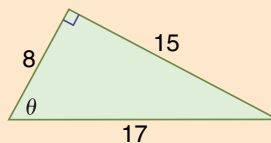
# 13:06 | Finding an Unknown Angle



Complete the ratios below for each triangle.



- 1  $\sin \theta =$
- 2  $\cos \theta =$



- 3  $\tan \theta =$
- 4  $\sin \theta =$

Given that  $\theta$  is acute, find  $\theta$  to the nearest degree, if:

- 5  $\tan \theta = 0.635$
- 6  $\sin \theta = 0.2135$
- 7  $\cos \theta = 0.0926$

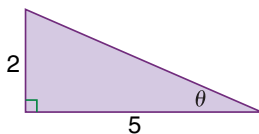
If  $0^\circ \leq A \leq 90^\circ$ , find  $A$  to the nearest minute if:

- 8  $\sin A = 0.52$
- 9  $\tan A = 2.673$
- 10  $\cos A = 0.7231$

We have already seen in 13:04 that a calculator can be used to find the size of an angle if the value of the trigonometric ratio is known.

## worked examples

- 1 Find the size of angle  $\theta$ .  
Answer to the nearest degree.



- 2 What angle, to the nearest minute, does the diagonal of a rectangle make with its length, if the dimensions of the rectangle are 12.6 cm by 8.9 cm?

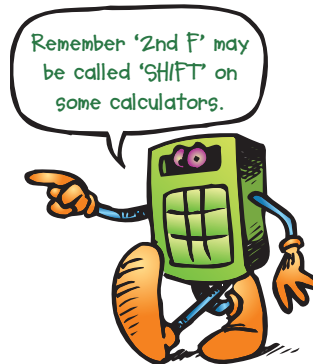
### Solutions

- 1 In the triangle,

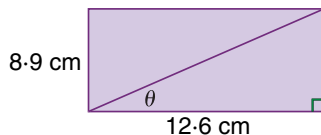
$$\begin{aligned} \tan \theta &= \frac{2}{5} \\ &= 0.4 \leftrightarrow \boxed{2\text{nd F}} \boxed{\tan} \boxed{0.4} \boxed{=} \end{aligned}$$

$$\therefore \theta = 21.801\ 409^\circ$$

so  $\theta = 22^\circ$  (to the nearest degree).



- 2 Let the required angle be  $\theta$ . Then:



$$\tan \theta = \frac{8.9}{12.6} \quad \boxed{2\text{nd F}} \boxed{\tan} \boxed{(} \boxed{8.9} \boxed{)} \boxed{12.6} \boxed{)} \boxed{=} \boxed{2\text{nd F}} \boxed{\text{DMS}}$$

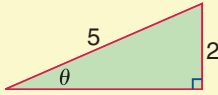
$$\therefore \theta = 35^\circ 14' 7.59''$$

$\therefore \theta = 35^\circ 14'$  (to the nearest minute).

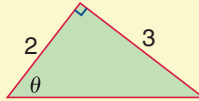
## Exercise 13:06

- 1** Find the size of the angle marked  $\theta$  in each triangle. Give your answers correct to the nearest degree.

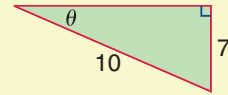
**a**



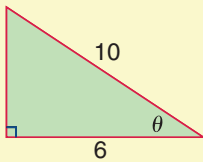
**b**



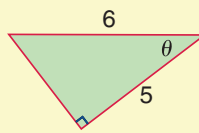
**c**



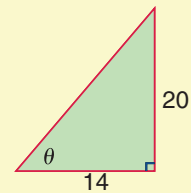
**d**



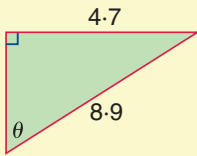
**e**



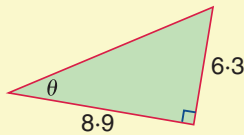
**f**



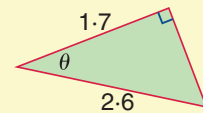
**g**



**h**

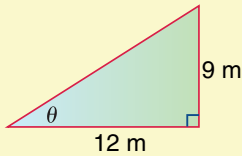


**i**

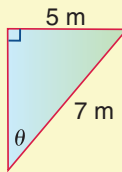


- 2** For each, find the size of  $\theta$  correct to the nearest minute.

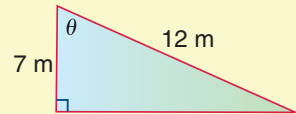
**a**



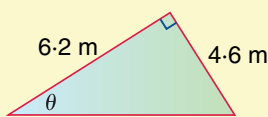
**b**



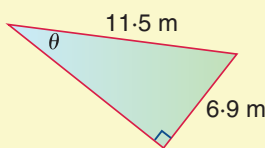
**c**



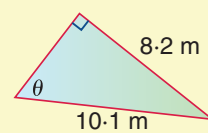
**d**



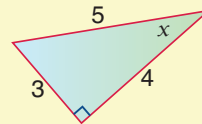
**e**



**f**

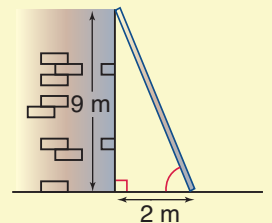


- 3** Use trigonometry to find  $x$  in three different ways.

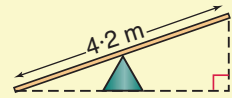


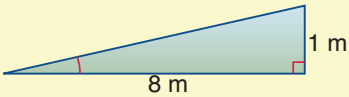
- 4** **a** In  $\triangle LMN$ ,  $\angle M = 90^\circ$ ,  $LN = 9.2$  m and  $LM = 8.2$  m. Find  $\angle L$ , to the nearest degree.  
**b** In  $\triangle PQR$ ,  $\angle R = 90^\circ$ ,  $PR = 6.9$  m and  $QR = 5.1$  m. Find  $\angle P$ , to the nearest minute.

- 5** **a** A ladder reaches 9 m up a wall and the foot of the ladder is 2 m from the base of the wall. What angle does the ladder make with the ground? (Answer correct to the nearest degree.)  
**b** What angle will a 5 m ladder make with the ground if it is to reach 4.4 m up a wall? (Answer correct to the nearest degree.)



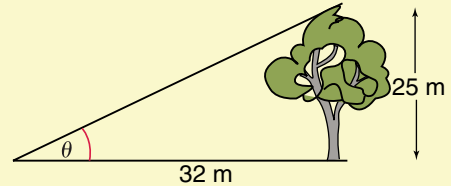
- 6** The beam of a see-saw is 4.2 m long. If one end is 1.2 m above the ground when the other end is resting on the ground, find the angle the beam makes with the ground, correct to the nearest degree.



- 7** 

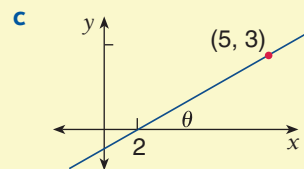
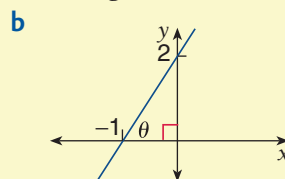
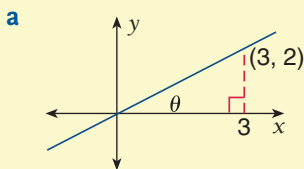
A road is inclined so that it rises 1 m for each horizontal distance of 8 m. What angle does the road make with the horizontal? (Answer correct to the nearest minute.)

- 8** At a certain time of the day, a tree 25 m high casts a shadow 32 m long. At this time of day, what angle do the rays of the sun make with the ground? (Answer correct to the nearest minute.)

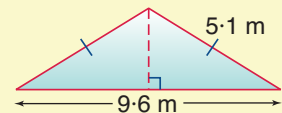


- 9** What angle does a diagonal of a rectangle make with each of the sides if the dimensions of the rectangle are 4.7 m by 3.2 m? (Answer correct to the nearest minute.)

- 10** Find the angle  $\theta$  in each of the following. (Answer correct to the nearest minute.)

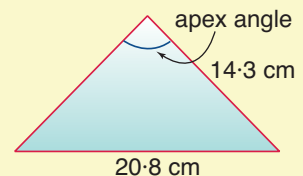


- 11** The cross-section of a roof is an isosceles triangle. Find the pitch of the roof (the angle it makes with the horizontal) if the width of the roof is 9.6 m and the length of one of the pitched sections is 5.1 m. Give your answer correct to the nearest minute.



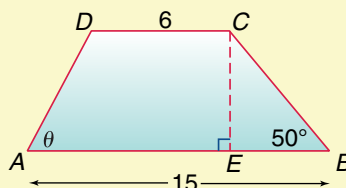
- 12** Find the size of the base angles of an isosceles triangle if the length of the base is 10 cm and the height is 8.4 cm. (Answer to the nearest minute.)

- 13** Find the apex angle of an isosceles triangle, if the length of each of the equal sides is 14.3 cm and the length of the base is 20.8 cm. Give your answer to the nearest minute.



- 14** The diagram shows a trapezium.

- a** If  $BC = 8$ , find  $\theta$ .  
**b** If  $CE = 8$ , find  $\theta$ .

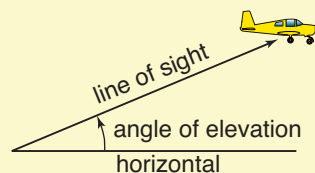


# 13:07 | Miscellaneous Exercises

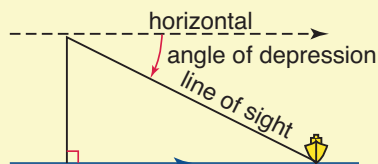
Before continuing with further trigonometric examples there is some general information that should be mentioned.

## Angles of elevation and depression

When looking upwards towards an object, the **angle of elevation** is defined as the angle between the line of sight and the horizontal.



When looking downwards towards an object, the **angle of depression** is defined as the angle between the line of sight and the horizontal.



## worked examples

- The angle of elevation of the top of a vertical cliff is observed to be  $23^\circ$  from a boat 180 m from the base to the cliff. What is the height of the cliff? (Answer correct to 1 decimal place.)
- An observer stands on the top of a 40-metre cliff to observe a boat that is 650 metres out from the base of the cliff. What is the angle of depression from the observer to the boat? (Answer to the nearest minute.)

## Solutions

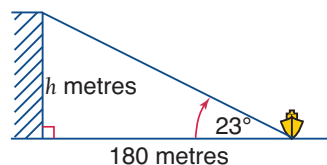
- For this example, the diagram would look like the one on the right.

Let the height of the cliff be  $h$  metres.

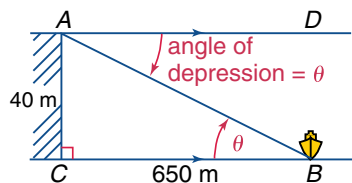
$$\text{Then: } \frac{h}{180} = \tan 23^\circ$$

$$\text{ie } h = (\tan 23^\circ) \times 180 \\ = 76.405\ 467 \text{ (from calculator)}$$

$\therefore$  Height of cliff = 76.4 m (to 1 decimal place).



2



Note: The angle of depression  $\angle DAB = \angle ABC$  (alternate angles and parallel lines).

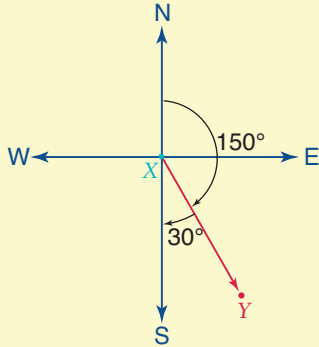
$$\tan \theta = \frac{40}{650} \quad \text{2nd F} \quad \tan \quad ( \quad 40 \quad ) \quad 650 \quad ) \quad = \quad \text{DMS}$$

$$\text{ie } \theta = 3^\circ 31' 17.23'' \\ = 3^\circ 31' \text{ (to the nearest minute).}$$



## Compass bearings

The direction of a point  $Y$  from an original point  $X$  is known as the **bearing** of  $Y$  from  $X$ . This is mainly expressed in one of two ways. Examine the diagram below.



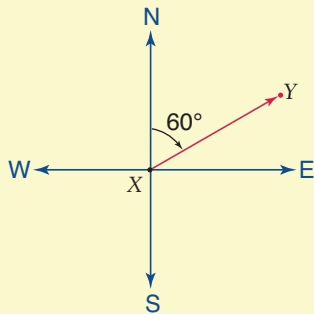
The bearing of  $Y$  from  $X$  can be given as:

- $150^\circ$  (the angle between the interval  $XY$  and the north line measured in a clockwise direction), or,
- south  $30^\circ$  east ( $S30^\circ E$ ).

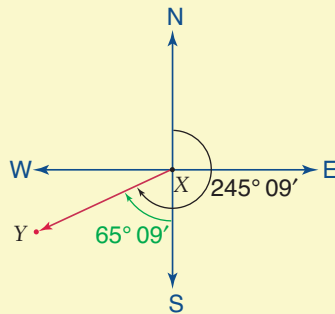
Sometimes, only letters are used. So SE (or south-east) is halfway between south ( $180^\circ$ ) and east ( $90^\circ$ ); that is,  $135^\circ$  or  $S45^\circ E$ .



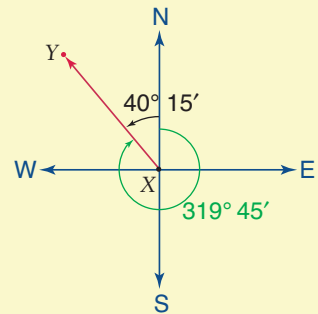
Other examples would look like these.



$060^\circ$  or  $N60^\circ E$



$245^\circ 09'$  or  $S65^\circ 09' W$



$319^\circ 45'$  or  $N40^\circ 15' W$

### worked examples

- If the town of Bartley is 5 km north and 3 km west of Kelly Valley, find the bearing of Bartley from Kelly Valley.
- Two people start walking from the same point. The first walks due east for 3.5 km and the second walks in the direction  $123^\circ$  until the second person is due south of the first person. How far did the second person walk (to the nearest metre)?



continued  $\rightarrow\rightarrow\rightarrow$

## Solutions

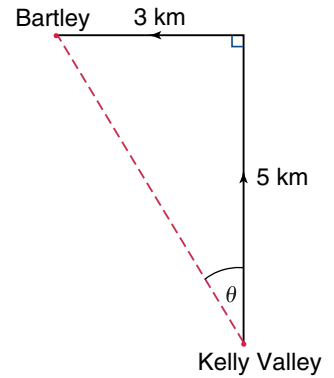
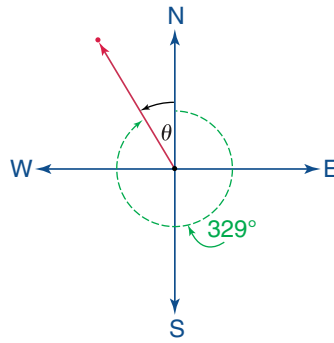
- 1 The diagram for this question would look like the one on the right.

Let the angle indicated in the diagram be  $\theta$ .

$$\begin{aligned}\text{Thus: } \tan \theta &= \frac{3}{5} \\ &= 0.6\end{aligned}$$

$$\text{So: } \theta = 31^\circ \text{ (to the nearest degree)}$$

So the bearing of Bartley from Kelly Valley would be  $N31^\circ W$  or simply  $329^\circ$ .



- 2 This diagram shows the information in the question above.

Since  $\angle SAB = \angle CBA$   
(alternate angles,  $AS \parallel CB$ )

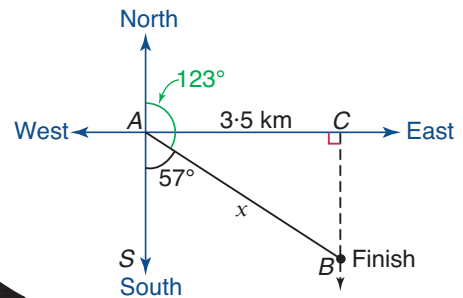
then  $\angle CBA = 57^\circ$

$$\text{So: } \frac{3.5}{x} = \sin 57^\circ$$

$$\begin{aligned}\text{ie } \frac{x}{3.5} &= \frac{1}{\sin 57^\circ} \\ x &= \frac{3.5}{\sin 57^\circ} \\ &= 4.173 \text{ km}\end{aligned}$$

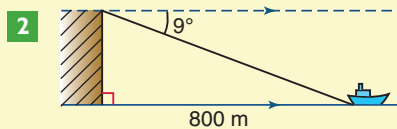
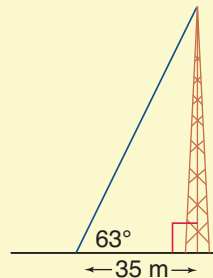
Press: 3.5   57

Check out this step!



## Exercise 13:07

- 1 The angle of elevation of the top of a tower from a point 35 m from the base of the tower was measured with a clinometer and found to be  $63^\circ$ . Find the height of the tower, correct to 1 decimal place.

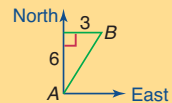


The angle of depression of a boat 800 m out to sea from the top of a vertical cliff is  $9^\circ$ . Find the height of the cliff, to the nearest metre.

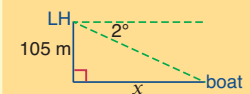
### Foundation Worksheet 13:07

#### Angles of elevation and depression, and bearings

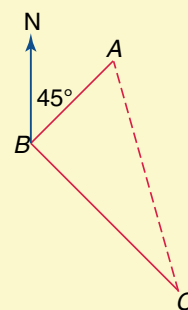
- 1 Find the bearing of B from A if B is 6 km north and 3 km east of A.



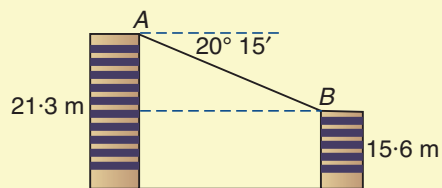
- 2 From a lighthouse 105 m above the sea the angle of depression of a boat is  $2^\circ$ . How far is the boat from the shore?



- 3** From the top of a cliff 72 m high, the angle of depression of a boat is  $12^{\circ}47'$ . How far is the boat from the base of the cliff? (Answer to the nearest metre.)
- 4** A vertical shadow stick has a height of 1.8 m. If the angle of elevation of the sun is  $42^{\circ}$ , what is the length of the shadow at that time, correct to 1 decimal place?
- 5** Find the angle of elevation of the top of a vertical tower from a point 25 m from its base, if the height of the tower is 40 m. (Answer to the nearest degree.)
- 6** From a lighthouse 70 m above sea level a ship, 1.2 km out to sea, is observed. What is the angle of depression from the lighthouse to the ship? (Answer to the nearest minute.)
- 7** A kite is on the end of a string 80 metres long. If the vertical height of the kite, above the ground, is 69 metres, find the angle of elevation of the kite from the person holding the string. (Assume the string is a straight line, and answer to the nearest minute.)
- 8** A cyclist travels 15 km in the direction  $N15^{\circ}27'E$ . How far has he travelled in a northerly direction (to the nearest metre)?
- 9** A ship sails from  $P$  to  $Q$  a distance of 150 km on a course of  $120^{\circ}30'$ . How far is  $P$  north of  $Q$ ? Also, how far is  $Q$  east of  $P$ ? (Answer to the nearest kilometre.)
- 10** Two towns,  $A$  and  $B$ , are 9 km apart and the bearing of  $B$  from  $A$  is  $320^{\circ}$ . Find how far  $B$  is west of  $A$  (to the nearest kilometre).
- 11** Two cars leave from the same starting point, one in a direction due west, the second in a direction with a bearing of  $195^{\circ}$ . After travelling 15 km, the first car is due north of the second. How far has the second car travelled (to the nearest kilometre)?
- 12** An aircraft flew 10 km south and then 6 km west. What is its bearing from its starting point? (Answer to the nearest degree.)
- 13**  $A$ ,  $B$  and  $C$  are three towns.  $A$  lies 7 km north-east of  $B$ , and  $B$  lies 12.5 km north-west of  $C$ . Find the bearing of  $A$  from  $C$ . Also, how far is  $A$  from  $C$ ? (Answer to the nearest metre.)
- 14** A ship is 5 nautical miles from a wharf on a bearing of  $321^{\circ}$ , and a lighthouse is 11.5 nautical miles from the wharf on a bearing of  $231^{\circ}$ . Find the bearing of the ship from the lighthouse. (Answer correct to the nearest minute.)
- 15** The bearings from a point  $P$  of two landmarks  $X$  and  $Y$  are  $35^{\circ}$  and  $125^{\circ}$  and their distances from  $P$  are 420 m and 950 m respectively. Find the bearing of  $Y$  from  $X$  (to the nearest minute).
- 16**  $X$  is due north of  $Y$  and 2 km distant.  $Z$  is due east of  $Y$  and has a bearing of  $S35^{\circ}12'E$  from  $X$ . How far, to the nearest metre, is  $Z$  from  $X$ ?

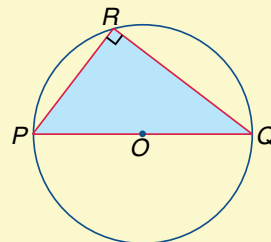


- 17** A wire is stretched from point  $A$  on the top of a building 21.3 m high, to point  $B$  on the top of a shorter building, 15.6 m high. The angle of depression from  $A$  to  $B$  is  $20^\circ 15'$ .

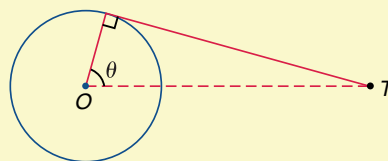


- a** What is the horizontal distance between the buildings (to the nearest centimetre)?  
**b** How long is the wire (to the nearest centimetre)?

- 18**  $PQ$  is a diameter of the circle, centre  $O$ , as shown with  $\angle PRQ = 90^\circ$ . If the radius of the circle is 6 cm, find, to the nearest millimetre, the length of the chord  $PR$ , given that  $\angle PQR = 40^\circ$ .

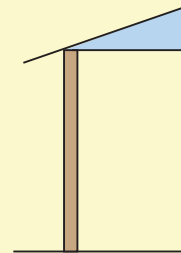


- 19** A tangent of length 16 cm is drawn to a circle of radius 7.5 cm from an external point  $T$ . What is the angle, marked  $\theta$  in the diagram, that this tangent subtends at the centre of the circle?



- 20** The diagonals of a rhombus are 11 cm and 7.6 cm. Find the angles, to the nearest degree, of the rhombus.  
**21** Find the acute angle, to the nearest minute, between the diagonals of a rectangle that has sides of 8 cm and 14 cm.

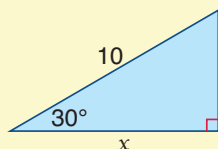
- 22** The eaves of a roof sloping at  $23^\circ$  overhang the walls, the edge of the roof being 75 cm from the top of the wall. The top of the wall is 5.4 metres above the ground. What is the height above the ground of the edge of the roof, to the nearest centimetre?



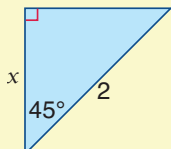
- 23** The arms of a pair of compasses are each 12 cm long. To what angle (to the nearest minute) must they be opened to draw a circle of 4 cm radius? How far from the paper will the joint be, if the compasses are held upright? (Answer to the nearest millimetre.)

- 24** Find the *exact* value of  $x$  in each of the following.

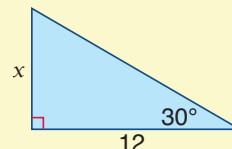
**a**



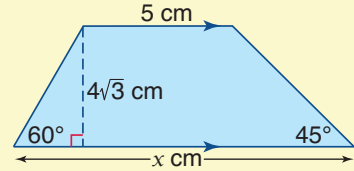
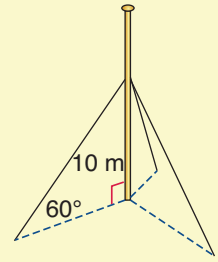
**b**



**c**



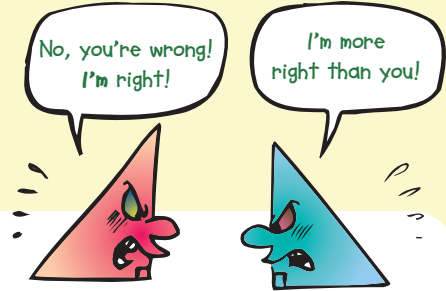
- 25 a** A rectangle is 10 cm long. The angle between the diagonal and the length is  $30^\circ$ . What is the exact area of the rectangle?
- b** A pole is to be supported by three guy wires. The wires are to be fixed 10 m from the base of the pole and must form an angle of  $60^\circ$  with the ground (which is horizontal). What will be the exact length of each guy wire?
- c** Find the exact value of  $x$  in the diagram.



## 13:08 | Problems Involving Two Right Triangles

Some problems can be solved by the consideration of two right-angled triangles within the problem.

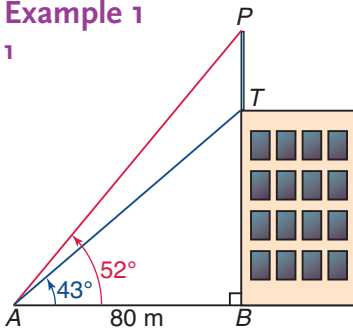
Examine the following two problems carefully and then attempt Exercise 13:08.



### worked examples

#### Example 1

1



A pole  $PT$  stands on the top of a building  $BT$ . From a point  $A$ , located 80 m from  $B$ , the angles of elevation of the top of the building and the top of the pole are  $43^\circ$  and  $52^\circ$  respectively. Find the height of the pole,  $PT$ , correct to the nearest metre.

#### Solution 1

- 1 To find the length of the pole  $PT$ , the lengths  $PB$  and  $TB$  are calculated and of course  $PT = PB - TB$ .

$$\text{In } \triangle PBA, \frac{PB}{80} = \tan 52^\circ$$

$$PB = 80 \tan 52^\circ$$

$$\text{In } \triangle TBA, \frac{TB}{80} = \tan 43^\circ$$

$$TB = 80 \tan 43^\circ$$

$$\begin{aligned} \text{Now } PT &= PB - TB \\ &= 80 \tan 52^\circ - 80 \tan 43^\circ \\ &= 80 (\tan 52^\circ - \tan 43^\circ) \\ &= 28 \text{ m (to the nearest metre)} \end{aligned}$$

continued  $\rightarrow\rightarrow\rightarrow$

### Example 2

$P$ ,  $Q$  and  $R$  are three villages.  $Q$  is 5 km and  $N25^\circ E$  from  $P$ .  $R$  is east of  $Q$  and is 6.7 km from  $P$ . What is the bearing of  $R$  from  $P$ , to the nearest degree?

### Solution 2

To find the bearing of  $R$  from  $P$ , we need to find the size of angle  $NPR$ . In  $\triangle NPR$  we know the length of  $PR$ , but we need to know one of the other sides, either  $NR$  or  $NP$ . Side  $NP$  can be calculated using  $\triangle NPQ$ .

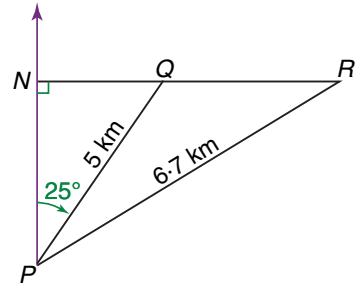
$$\text{In } \triangle NPQ: \quad \frac{NP}{5} = \cos 25^\circ$$

$$\text{ie} \quad NP = 5 \cos 25^\circ$$

$$\begin{aligned} \text{In } \triangle NPR: \cos \angle NPR &= \frac{NP}{6.7} \\ &= \frac{5 \cos 25^\circ}{6.7} \\ &= 0.676\ 349 \end{aligned}$$

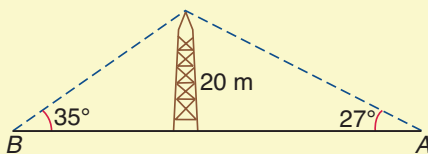
$$\therefore \angle NPR = 47^\circ \text{ (to the nearest degree)}$$

$\therefore$  The bearing of  $R$  from  $P$  is  $N47^\circ E$ .

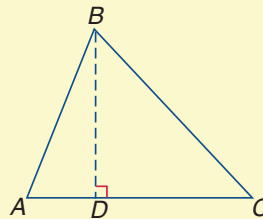


## Exercise 13:08

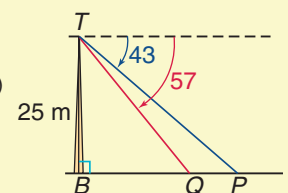
- 1** The top of a 20-metre tower is observed from two positions,  $A$  and  $B$ , each in line with, but on opposite sides of, the tower. If the angle of elevation from  $A$  is  $27^\circ$  and from  $B$  is  $35^\circ$ , how far is point  $A$  from point  $B$  (to the nearest metre)?



- 2** In triangle  $ABC$ ,  $BD$  is perpendicular to  $AC$ . Given that  $AB = 13$  m,  $BD = 11$  m and  $DC = 10$  m, find, to the nearest degree, the size of angle  $ABC$ .

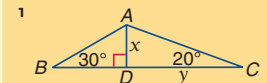


- 3** Two points,  $P$  and  $Q$ , are in line with the foot of a tower 25 m high. The angle of depression from the top of the tower to  $P$  is  $43^\circ$  and to  $Q$  is  $57^\circ$ . How far apart are the points? (Answer to the nearest metre.)

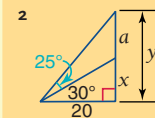


### Foundation Worksheet 13:08

#### Problems with more than one triangle

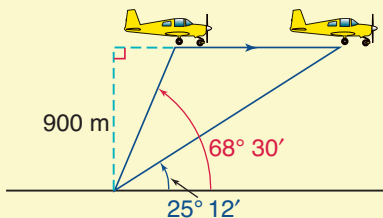


- a Use  $\triangle ABD$  to find  $x$ .  
b Use  $\triangle ADC$  to find  $y$ .



Use the fact that  $a = y - x$  to find the value of  $a$ .

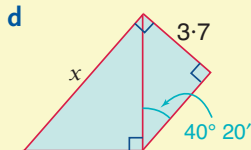
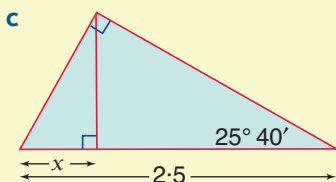
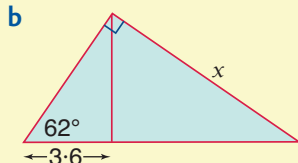
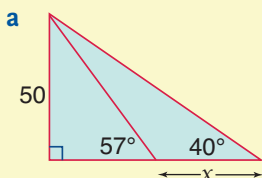
4



A plane is flying at an altitude of 900 m. From a point  $P$  on the ground, the angle of elevation to the plane was  $68^{\circ}30'$  and 20 seconds later the angle of elevation from  $P$  had changed to  $25^{\circ}12'$ . How far had the plane flown in that time, and what was its speed, to the nearest kilometre per hour? (Find the distance to the nearest metre.)

5

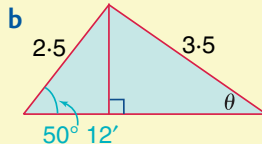
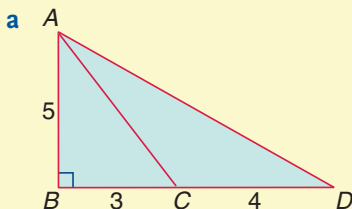
Find  $x$  in each diagram. Give answers correct to 2 decimal places.



Find a different side first.



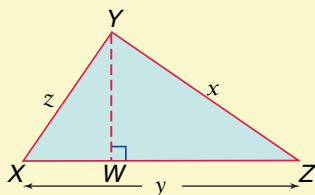
6



Find  $\angle CAD$  to the nearest minute. Find  $\theta$  to the nearest minute.

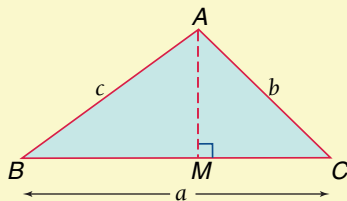
7

- a From  $\triangle XWY$ , show that  $XW = z \cos X$ .  
 b From  $\triangle ZWY$ , show that  $ZW = x \cos Z$ .  
 c Hence show that  $y = z \cos X + x \cos Z$ .



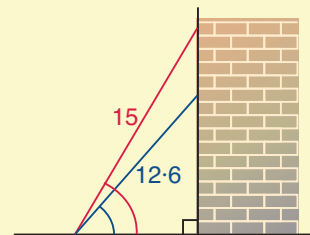
8

- a Show that  $AM = c \sin B$ .  
 b Hence show that the area of  $\triangle ABC = \frac{1}{2} ac \sin B$ .

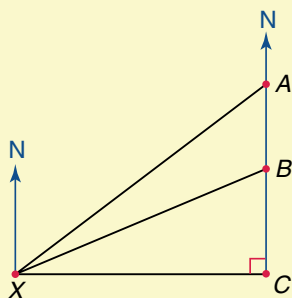


9

Two ladders are the same distance from the base of a wall. The longer ladder is 15 m long and makes an angle of  $58^{\circ}$  with the ground. If the shorter ladder is 12.6 m long, what angle does it make with the ground? (Answer to the nearest degree.)

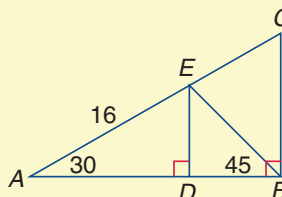


10



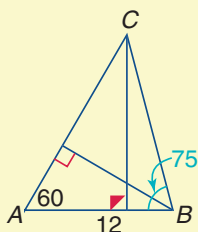
$A$ ,  $B$  and  $C$  are three towns where  $A$  and  $B$  are due north of  $C$ . From a position  $X$  on a map,  $A$  has a bearing of  $N27^\circ E$  and  $B$  has a bearing of  $N67^\circ E$ . Town  $C$  is due east of  $X$  and  $7.5$  km from it. Find the distance, correct to 1 decimal place, between  $A$  and  $B$ .

11 Find the exact value of  $CE$  given that  $AE = 16$ .



12 In  $\triangle ABC$ ,  $AB = 12$ ,  $\angle CAB = 60^\circ$  and  $\angle CBA = 75^\circ$ . Find as exact values:

- a  $AC$
- b  $BC$
- c area of  $\triangle ABC$



### Fun Spot 13:08 | What small rivers flow into the Nile?

Work out the answer to each question and put the letter for that part in the box that is above the correct answer.

What are the exact values of:

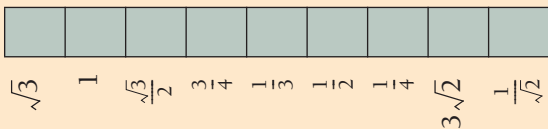
I  $\cos 60^\circ$                       E  $(\cos 30^\circ)^2$

L  $(\sin 30^\circ)^2$                   J  $\tan 60^\circ$

V  $\sin 60^\circ$                         N  $(\tan 30^\circ)^2$

S  $\sin 45^\circ$                         E  $\frac{3}{\cos 45^\circ}$

U  $(\sin 30^\circ)^2 + (\cos 30^\circ)^2$



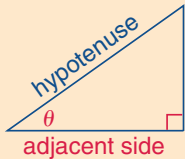
Challenge worksheet 13:08 Three-dimensional problems



## Mathematical Terms 13

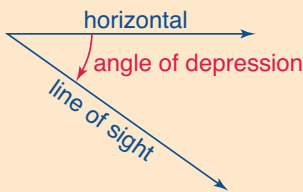
### adjacent side (to a given angle)

- The side of a triangle which together with the hypotenuse forms the arms of a given angle.



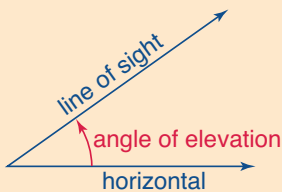
### angle of depression

- When looking down, the angle between the line of sight and the horizontal.



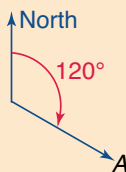
### angle of elevation

- When looking up, the angle between the line of sight and the horizontal.



### bearing

- An angle used to measure the direction of a line from north.
- Bearings can be recorded in two ways.  
eg  $120^\circ$  or  $S60^\circ E$



### cosine ratio (of an angle $\theta$ )

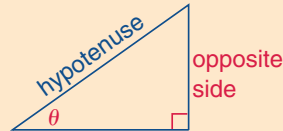
- The ratio  $\frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$
- Abbreviated to  $\cos \theta$ .

### hypotenuse

- The longest side in a right-angled triangle.
- The side which is not one of the arms of the right-angle in a right-angled triangle.

### opposite side (to a given angle)

- The side of a triangle which is not one of the arms of the given angle.



### similar triangles

- Two triangles that have the same shape but a different size.
- Triangles that can be changed into each other by either an enlargement or reduction.
- Triangles that have matching angles equal.
- Triangles where the ratio of matching sides is constant.

### sine ratio (of an angle $\theta$ )

- The ratio  $\frac{\text{side opposite angle } \theta}{\text{hypotenuse}}$
- Abbreviated to  $\sin \theta$ .

### tangent ratio (of an angle $\theta$ )

- The ratio  $\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$
- Abbreviated to  $\tan \theta$ .

### trigonometric (trig.) ratios

- A collective name for different ratios of the side lengths of right-angled triangles.
- The ratios have constant values for any particular angle.

### trigonometry

- A branch of mathematics, part of which deals with the calculation of the sides and angles of triangles.



1 The trigonometric ratios  
2 Finding sides  
3 Finding angles

4 Bearings 1  
5 Bearings 2

Mathematical terms 13

